RESTRICTION OF p-MODULAR REPRESENTATIONS OF p-ADIC GROUPS TO MINIMAL PARABOLIC SUBGROUPS

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This manuscript reports on an ongoing joint work with Julien Hauseux, on which is based the talk I gave on January 2020 at the RIMS workshop "Analytic, geometric and p-adic aspects of automorphic forms and L-functions". I thank Shunsuke Yamana for inviting me to participate to this very nice workshop and for the invitation to speak there. For more details on the work described below, e.g. statements in their full generality or further details of proofs, please refer to our paper with Hauseux [3]. For any question or comment on this work, please feel free to email me (Ramla.Abdellatif@u-picardie.fr).

1. Context and motivation : the GL_2 case

1.1. **What is the context?** Let p be a prime number and F be a non-Archimedean local field with finite residue field k_F of characteristic p (hence F is either a finite extension of the field of p-adic numbers $\mathbb{Q}_p$, or a Laurent series field with coefficients in k_F). Let $\mathcal{O}_F$ be its ring of integers and $\mathfrak{p}_F$ be its maximal ideal. Given a connected reductive group $\mathcal{G}$ defined over F, a long-standing question in representation theory has been to classify all (isomorphism classes of) smooth[1] representations of the topological group $G := \mathcal{G}(F)$ over any given algebraically closed field C of characteristic p. This question is at the heart of current exciting developments in number theory, since it is directly (motivated by and) connected to p-modular and p-adic Langlands correspondences, as well as to congruences between modular forms or Galois representations.

Even classifying **irreducible** smooth representations of G in characteristic p is a hard challenge, and not much is known so far. Thanks to the work of many people, as [5, 6, 7] for GL_2, or [1, 4, 11, 13] for more general $\mathcal{G}$, we know that such representations separate between non-supercuspidal and supercuspidal representations: the former are quite well understood (see [2] when $\mathcal{G}$ is of relative rank 1, or [4] for arbitrary $\mathcal{G}$), while the latter are still really mysterious. Note that in this context, we do not have access to the usual tools of harmonical analysis (such as Haar measures, that do not exist in natural characterstic) nor to classical type theory (as everything would then be of level 0). Also recall that, in the Langlands philosophy, supercuspidal representations of G are conjecturally the counterpart of irreducible Galois representations, hence they are of great importance.

So far, the full classification of supercuspidal representations of G is only known in three cases [1, 7, 11], which all require $F = \mathbb{Q}_p$ and $\mathcal{G}$ to be of relative rank 1 over $\mathbb{Q}_p$. Apart from this, very few things are known about supercuspidal representations, even for $G = GL_2(F)$, and most of the results proven for now only show that going to arbitrary F

[1] Recall that a representation of G is *smooth* when every vector has open stabiliser in G.

will certainly be extremely difficult, see for instance in [8, 10, 12, 15]. Any positive result about supercuspidal representations for arbitrary F would hence be warmly welcome.

1.2. **What is the motivation?** Let us start by focusing on the case where $\mathcal{G} = \mathrm{GL}_2$ and no assumption is done on F. If B denotes the subgroup of upper triangular matrices in $G = \mathrm{GL}_2(F)$, then we know from [5, 6] that (isomorphism classes of) irreducible smooth representations of G split into four families (and that there is no non-trivial isomorphism among non-supercuspidal representations up to twists by smooth characters of G):

- one-dimensional representations $\chi \circ \det$ with $\chi : F^\times \to C^\times$ a smooth character;
- *principal series representations*, which are parabolically induced representations of the form $\mathrm{Ind}_B^G(\chi_1 \otimes \chi_2)$ where $\chi_1, \chi_2 : F^\times \to C^\times$ are distinct smooth characters;
- *special series representations* $\mathrm{St}_G \otimes (\chi \circ \det)$, where St_G denotes the Steinberg representation and $\chi : F^\times \to C^\times$ is a smooth character;
- supercuspidal (or *supersingular*[2]) representations.

As already mentioned above, the last family remains very mysterious when $F \neq \mathbb{Q}_p$, and new tools and ideas are required to make any progress. Among the numerous people who work hard on this problem, Paškūnas studied the structure of $C[B]$-module carried by these representations (now seen as representations of B) and proved the following result [14, Theorem 1.1], where $\overline{\mathbb{F}}_p$ denotes an algebraic closure of the residue field of F.

Theorem 1.1 (Paškūnas 2007). *Let π and π' be smooth representations of G over $\overline{\mathbb{F}}_p$. Assume that π is irreducible and has central character.*

(1) *The $\overline{\mathbb{F}}_p[B]$-module $\pi|_B$ is of finite length at most 2, with equality iff π is a principal series representation.*
(2) *If π is not a special series representation, then the canonical restriction map*
$$\mathrm{Hom}_G(\pi, \pi') \xrightarrow{\simeq} \mathrm{Hom}_B(\pi|_B, \pi'|_B)$$
is an isomorphism of $\overline{\mathbb{F}}_p$-vector spaces.
(3) *The restriction map (as in the previous statement) induces an isomorphism of $\overline{\mathbb{F}}_p$-vector spaces*
$$\mathrm{Hom}_G(\mathrm{Ind}_B^G(\mathbf{1}), \pi') \xrightarrow{\simeq} \mathrm{Hom}_B(\mathrm{St}_G|_B, \pi'|_B) \ .$$

In particular, this theorem shows that supercuspidal representations of $\mathrm{GL}_2(F)$ remain irreductible when seen as representations of B, and that their isomorphism classes (as representations of G) are completely determined by their isomorphism classes as representations of B, which seem more accessible (see for instance [16]). Note that these results can moreover be lifted to characteristic 0 to get a p-adic counterpart relative to unitary Banach representations of $\mathrm{GL}_2(F)$, see [14, Section 6] for precise statements.

One of the motivations of our work with Hauseux is to answer the following question: to what extent do the previous theorem hold for other groups than $\mathrm{GL}_2(F)$?

[2] I do not want to define supersingularity as it would require a lot of efforts that are unnecessary here. Let me just point out that the fact that being supercuspidal is equivalent to being supersingular is a very difficult theorem, see the references given in the bibliography for more information.

1.3. **Beyond $\mathrm{GL}_2(F)$: some limits of Paškūnas' proof.** Paškūnas' proof of Theorem 1.1 distinguish between non-supercuspidal and supercuspidal representations, the latter being the most interesting part of the result. In the supercuspidal case, the proof of the theorem heavily relies on the following lemma, where $I(1)$ is the standard pro-p-Iwahori subgroup[3] of G, ϖ_F is a fixed uniformiser of F and $[.] : k_F \to \mathcal{O}_F$ is the Teichmüller lift.

Lemma 1.2 (Paškūnas 2007). *Set* $\Sigma := \sum_{\lambda \in k_F} \begin{pmatrix} \varpi_F & [\lambda] \\ 0 & 1 \end{pmatrix}$ *and* $s := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.
Let π be a smooth representation of G and let $v \in \pi^{I(1)}$ be a non-zero vector[4]*.*

(1) *If $\Sigma v = 0$, then* $sv = -\sum_{\lambda \in k_F^\times} \begin{pmatrix} -\varpi_F[\lambda^{-1}] & 1 \\ 0 & \varpi_F^{-1}[\lambda] \end{pmatrix} v$ *belongs to $\langle B \cdot v \rangle \subset \pi$.*
(2) *If π is supercuspidal (hence irreducible by definition) and if $\langle \mathrm{GL}_2(\mathcal{O}_F) \cdot v \rangle \subset \pi$ is irreducible (as representation of $\mathrm{GL}_2(\mathcal{O}_F)$), then there exists a positive integer n such that $\Sigma^n v = 0$.*

An important point here is that the matrices of B appearing in the decomposition of sv in $\langle B \cdot v \rangle$ do not depend on the element v. Another keystone to prove the second assertion is that for $\mathrm{GL}_2(F)$, supersingularity (that is equivalent here to supercuspidality) is by definition *very* closely related to the vanishing of Σ (see [14, Lemma 3.1] for more information). This last feature fails when one is interested in other groups than $\mathrm{GL}_2(F)$ (see [4]), even for rank 1 groups as $\mathrm{GL}_2(D)$ with D a division algebra over F (see [13]). Moreover, the proof of Paškūnas uses in a non-trivial way several other specificities of $\mathrm{GL}_2(F)$, such as some combinatorial correspondences related to the Bruhat-Tits tree, or very explicit matrix calculations that cannot be carried out similarly in other settings, and distinguish between cases that seem a bit artificial at first (depending on the dimension of σ, with the notation of [14]). One can transfer Paškūnas' proof to some groups closely related to $\mathrm{GL}_2(F)$ (as $\mathrm{SL}_2(F)$, in an unpublished work of the author): it only brings further technicalities and does not help to find a natural way to generalise Theorem 1.1.

2. Main results of our work

2.1. **Notations.** From now on, $\mathcal{G}$ denotes a connected reductive group of relative rank 1 over F. For the simplicity of the exposition, I assume that G is quasi-split over F, thought results and proofs actually hold without this assumption. Let S be a maximal split torus in G and T be its centraliser in G. Let $W = \langle w_0 \rangle \simeq \mathbb{Z}/2\mathbb{Z}$ denote the Weyl group of G. Let $B = TU$ be a Borel subgroup[5] with unipotent radical U and $\overline{B} = B^{w_0} = T\overline{U}$ be its opposite Borel subgroup (with respect to T).

Fix a special point x_0 in the apartment corresponding to S in the semisimple Bruhat-Tits building $\mathcal{X}$ of G and let K be its stabiliser in G : it is a special parahoric subgroup of G and we write $K(1)$ for its pro-p-radical. As K is special, one can choose a representative of w_0 (that will be denoted w_0 again) in $N \cap K$, where N is the normaliser of S in G.

[3]Recall that $I(1)$ denotes the subgroup of matrices in $\mathrm{GL}_2(\mathcal{O}_F)$ whose reduction modulo $\mathfrak{p}_F$ is upper-triangular and unipotent.

[4]Such a non-sero $I(1)$-invariant vector always exists by [6, Lemma 1].

[5]In general, B is a minimal parabolic subgroup of G but its Levi part is not necessarily a torus.

Now fix an alcove of $\mathcal{X}$ based at x_0. Its pointwise stabiliser I is called a *standard Iwahori subgroup* of G, and the pro-p-radical $I(1)$ of I is a *standard pro-p-Iwahori subgroup* of G.

Let U_0 be a fixed open compact subgroup of $U \cap K$ and let $T^+ := \left\{t \in T \mid t^{-1}U_0 t \subset U_0\right\}$ be the monoid of elements in T that contract U_0. It contains a distinguished element t_0 that can be easily defined in the quasi-split case since we can set $t_0 = \lambda(\varpi_F)$, where λ is a generator of the group $X_*(S)$ of algebraic cocharacters of S. In particular, we hence have $S/S \cap K \simeq t_0^{\mathbb{Z}}$.

2.2. **Statement of the main results.** Our first result consist in a suitable generalisation of Paškūnas' key lemma for rank 1 groups. Its statement is very close to Paškūnas' original lemma, up to the definition of the operator replacing Σ, which is more algebraic and less combinatorial. Nevertheless, note that we naturally recover Lemma 1.2 when $\mathcal{G} = \mathrm{GL}_2$: by taking $U_0 = U \cap \mathrm{GL}_2(\mathcal{O}_F)$, $t_0 = \begin{pmatrix} \varpi_F & 0 \\ 0 & 1 \end{pmatrix}$ and $w_0 = s$, one gets $\Sigma_0 = \Sigma$.

Lemma 2.1 (Abdellatif-Hauseux 2019)**.** *Let π be a smooth representation of G over $\overline{\mathbb{F}}_p$. Set* $\Sigma_0 := \sum_{u \in U_0/t_0^{-1}U_0 t_0} u t_0$.

(1) *If $v \in \pi^{I(1)}$ satisfies $\Sigma_0 v = 0$, then $w_0 \cdot v$ belongs to $\langle B \cdot v\rangle$ and the form of its decomposition along B does not depend on the choice of v.*
(2) *If π is an (irreducible) admissible supercuspidal representation of G, then:*

$$\forall\ v \in \pi^{U_0},\ \exists\ n \geq 1 \mid \Sigma_0^n v = 0\ .$$

A very satisfying point is that our proof does not require any strange distinction as Paškūnas' one on the dimension of σ. This is pleasant as it gives a uniform argument even in the GL_2 case. Also note that it requires a bit of care to get the independence of the decomposition along B, as one must check first that $\pi^{I(1)}$ is actually T-stable, which is true by Iwahori decomposition (used with the suitable order on factors).

Once this key lemma is proven, we can show the following theorem, which basically states that Theorem 1.1 holds for relative rank 1 groups.

Theorem 2.2 (Abdellatif-Hauseux 2019)**.** *Let π and π' be smooth representations of G over $\overline{\mathbb{F}}_p$. Assume that π is irreducible and admissible.*

(1) *The $\overline{\mathbb{F}}_p[B]$-module $\pi|_B$ is of finite length at most 2, with equality iff π is a principal series representation.*
(2) *If π is not a special series representation, then the canonical restriction map*

$$\mathrm{Hom}_G(\pi, \pi') \xrightarrow{\simeq} \mathrm{Hom}_B(\pi|_B, \pi'|_B)$$

is an isomorphism of $\overline{\mathbb{F}}_p$-vector spaces.
(3) *The restriction map (as in the previous statement) induces an isomorphism of $\overline{\mathbb{F}}_p$-vector spaces*

$$\mathrm{Hom}_G(\mathrm{Ind}_B^G(\mathbf{1}), \pi') \xrightarrow{\simeq} \mathrm{Hom}_B(\mathrm{St}_G|_B, \pi'|_B)\ .$$

Before I explain the main ideas behind the proofs of these results, I want to do some remarks about these statements and their importance.

- The admissibility assumption made on π is here to ensure that the equivalence between supercuspidality and supersingularity is valid [4, Theorem VI.2]. When one drops the admissibility assumption, the key lemma (and hence the theorem) hold for supersingular representations, but the latter may *a priori* not be enough to cover all supercuspidal representations. (A hard open question is to determine to what extent supersingularity and supercuspidality are actually equivalent.)
- The major novelty here is the supercuspidal case: indeed, the structure of $\overline{\mathbb{F}}_p[B]$-modules carried by non-supercuspidal representations of G when $\mathcal{G}$ is quasi-split of rank 1 was already known from one of my previous work [2]. Nevertheless, I should precise that in [3], we do not assume any splitting assumption on $\mathcal{G}$, hence we also prove new complete results in the non-supercuspidal case, as the latter was not fully understood when $\mathcal{G}$ is non-split (besides some partial results of Ly [13] for $\mathrm{GL}_2(D)$, nothing was done in that direction).
- Let me emphasize once more the fact that a very interesting feature of our work is that it gives a nice interpretation of Paškūnas' original calculation for $\mathrm{GL}_2(F)$, and makes this special case fit into the wider picture of relative rank 1 groups. This gives the hope that similar results should hold for higher rank groups, as $\mathrm{GL}_n(F)$ for $n \geq 3$ for instance (which is a part of a work in progress).

3. Main tools and ideas of proofs

In this last section, I want to introduce the main ideas and tools that allowed us to prove Lemma 2.1 and Theorem 2.2. The upshot is as follows.

- The non-supercuspidal case is not the most exciting part: Vignéras did most of the job for split groups in [17], I made it for quasi-split groups of rank 1 in [2], so our job here was basically to define the correct setting so that non-split groups also fit in. It is technical but does not require new deep ideas.
- The operator Σ that shows up in the key lemma (Lemma 2.1) actually transfers the value of the ordinary parts functor, as defined by Emerton (see the next subsection for a quick overview, and [9] for a complete lecture), and to be supersingular (or finite-dimensional, see below) amounts to have null ordinary parts.
- Computing ordinary parts for rank 1 groups is not too difficult...

3.1. **Recollection on ordinary parts.** The references for the content of this subsection are [9] and [18]. Note that it holds for $\mathcal{G}$ a connected reductive group of arbitrary rank. Let P be a parabolic subgroup of G and $P = LU$ be a Levi decomposition of P. For any subgroup Γ of G, denote by $\mathrm{Mod}_\Gamma^\infty$ the category of smooth representations of Γ over $\overline{\mathbb{F}}_p$. We know from Frobenius reciprocity that the restriction functor $\mathrm{Mod}_G^\infty \to \mathrm{Mod}_L^\infty$ is left-adjoint to the parabolic induction functor Ind_P^G:

$$\forall\, \sigma \in \mathrm{Mod}_L^\infty, \forall\, \pi \in \mathrm{Mod}_G^\infty,\ \mathrm{Hom}_G(\pi, \mathrm{Ind}_P^G(\sigma)) \simeq \mathrm{Hom}_L(\pi|_L, \sigma)\ .$$

(The inflation from L to P is naturally obtained by letting U act trivially.) This adjunction property is useful to study (irreducible) subrepresentations of $\mathrm{Ind}_P^G(\sigma)$, which are by definition non-supercuspidal representations of G. Similarly, it would be nice to have a right adjoint for this restriction functor, as it would help to identify quotients of

$\mathrm{Ind}_P^G(\sigma)$, hence to get a better grasp on supercuspidal representations (by playing with right and left adjunction properties).

In [9], Emerton defines this right adjoint functor, called the *(P-)ordinary parts functor*, as follows. (Note that [9] requires F to be of characteristic 0, but Vignéras' later work [18] ensures that everything actually makes sense for arbitrary F). As before, let U_0 be an open compact subgroup of $U \cap K$ and L^+ be the monoid of elements in L that contract U_0. Following [9, Definition 3.1.3], any smooth representation π of G over $\overline{\mathbb{F}}_p$ comes with a *Hecke action* of L^+ on π^{U_0}, namely:

$$\forall\, t \in L^+,\ \forall\, v \in \pi^{U_0},\ t \star v := \sum_{u \in U_0/t^{-1}U_0 t} utv\ .$$

As in [9, Definition 3.1.9], we set $\mathrm{Ord}_P(\pi) := \mathrm{Hom}_{\overline{\mathbb{F}}_p[L^+]}(\overline{\mathbb{F}}_p[L], \pi^{U_0})_{L\text{-finite}}$, which is an object of Mod_L^∞. It is not too difficult to check that this definition does not depend on the choice of U_0, and that we actually get a functor $\mathrm{Ord}_P : \mathrm{Mod}_G^\infty \to \mathrm{Mod}_L^\infty$. The main result is the following theorem, which gives the adjunction formula we hoped for, provided we consider induction from the **opposite** parabolic subgroup when using P-ordinary parts.

Theorem 3.1 (Main Adjunction Formula, Emerton 2010-Vignéras 2016)**.** *Let σ be an admissible smooth representation of L and π be a smooth representation of G. If $\overline{P}$ is the opposite parabolic subgroup to P relatively to L, then:*

$$\mathrm{Hom}_G(\mathrm{Ind}_{\overline{P}}^G(\sigma), \pi) \xrightarrow{\simeq} \mathrm{Hom}_L(\sigma, \mathrm{Ord}_P(\pi))$$

As a consquence, we get that an irreducible admissible smooth representation π of G is right cuspidal (i.e. either finite-dimensional or supercuspidal) iff $\mathrm{Ord}_P(\pi) = 0$ for any proper parabolic subgroup P of G.

3.2. **Application to the rank 1 (quasi-split) case.** Now assume that $\mathcal{G}$ has relative rank 1 over F and use the notations of Section 2. One can then see that the operator Σ_0 of Lemma 2.1 is nothing but the Hecke action of t_0 on π^{U_0}, which contains $\pi^{I(1)}$ as $U_0 \subset U \cap K \subset I(1)$. The vanishing property required in this lemma is hence naturally connected to the vanishing of some ordinary parts. More precisely, we have the following result, which comes from a quite straightforward calculation.

Proposition 3.2 (Abdellatif-Hauseux 2019)**.** *Let π be an irreducible smooth representation of G. Then*

$$\mathrm{Ord}_B(\pi) \simeq \overline{\mathbb{F}}_p[X^{\pm 1}] \otimes_{\overline{\mathbb{F}}_p[X]} \pi^{U_0}\ ,$$

where X corresponds to the Hecke action of t_0 on π^{U_0}.

As expected, this shows that the vanishing of $\mathrm{Ord}_B(\pi)$ corresponds to the fact that t_0 acts locally nilpotently on π_0^U, hence the proof of Lemma 2.1 for supercuspidal representations is quite straightforward (up to some subtleties pointed out earlier). Let me close this exposition by a sketch of the proof of Theorem 2.2 in the supersingular case.

- First, I explain how to prove the irreducibility of $\pi|_B$ when π is supersingular (i.e. supercuspidal as π is assumed to be admissible in the statement of Theorem 2.2). Let v be a non-zero vector of π. By smoothness of π and Iwahori decomposition relative to $I(1)$, we get that $\langle B \cdot v\rangle \cap \pi^{I(1)}$ is non-zero (since we actually have $\langle I(1)t_0^k v\rangle \subset \langle (B \cap I(1))t_0^k v\rangle$ for any integer k). Note that this first step is not straightforward at all, and requires some arguments to prove that this intersection contains some $t_0^k v$ for a well-chosen $k \in \mathbb{Z}$.

Now let x_0 be a non-zero element of $\langle B \cdot v\rangle \cap \pi^{I(1)}$. The second statement of Lemma 2.1 ensures the existence of a minimal $n \geq 1$ such that $x_1 := t_0^{n-1} x_0$ is non-zero and $\Sigma_0 x_1 = 0$. This last condition implies, thanks to the first statement of Lemma 2.1, that $w_0 x_1$ belongs to $\langle B\cdot x_1\rangle \subset \langle B\cdot x_0\rangle$. By Birkhoff decomposition, we now get that $\pi = \langle G \cdot x_1\rangle = \langle B \cdot x_1\rangle \subset \langle B \cdot x_0\rangle \subset \langle B \cdot v\rangle \subset \pi$, where the first and last equalities come from the irreducibility of π. This proves in particular that $\pi = \langle B \cdot v\rangle$ for any non-zero element v of π, and π is hence an irreducible representation of B.

- Now I explain how to get the assertion on the Hom spaces. Let π' be any smooth representation of G, and still assume that π is supersingular (and irreducible). The injectivity of the canonical linear map $\mathrm{Hom}_G(\pi, \pi') \to \mathrm{Hom}_B(\pi, \pi')$ is clear, so let us focus on its surjectivity. Let $\Phi \in \mathrm{Hom}_B(\pi, \pi')$ be a non-zero homomorphism: as π is irreducible, Φ must be injective. To prove that Φ is actually G-equivariant, pick a non-zero vector $v \in \pi^{I(1)}$, then a non-zero vector $v_0 \in \langle B \cdot v\rangle \cap \pi^{I(1)}$, which exists by what we did above to prove the irreducibility of $\pi|_B$. By a classical smoothness and filtration argument, one checks that $\Phi(v_0)$ is a non-zero $I(1)$-invariant vector of π'. More generally, set $v_{n+1} := \Sigma_0 v_n$ for any $n \geq 0$: since Σ_0 and Φ commute (by B-equivariance of Φ), one can prove by induction on n that v_n and $\Phi(v_n)$ are $I(1)$-invariant vectors (respectively of π and π'). But the second statement of Lemma 2.1 applied to v_0 ensures the existence of a minimal integer $n_0 \geq 1$ such that $v_{n_0+1} = 0$, hence such that $w_0 v_{n_0} \in \langle B\cdot v_{n_0}\rangle$ and $w_0\Phi(v_{n_0}) \in \langle B\cdot \Phi(v_{n_0})\rangle$ with a decomposition along B that does not depend on the chosen vector. In particular, as Φ is B-equivariant, this shows that $w_0\Phi(v_{n_0}) = \Phi(w_0 v_{n_0})$, hence that $g\Phi(v_{n_0}) = \Phi(g v_{n_0})$ for any $g \in G$ (by Birkhoff decomposition). By minimality of n_0, v_{n_0} is non-zero, hence we have $\pi = \langle G \cdot v_{n_0}\rangle$ by irreducibility of π, and Φ is actually G-equivariant, as expected.

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